1. Find and fix any [logic] errors in the following function. (Performance need not be a concern.)

```
long fib(short n)
{
    return fib(n-1) + fib(n-2);
}
```

2. There are 3 rules to recursion and a strong guideline. Explain each briefly.

3. Describe the general principles of back-tracking.

4. What is 'tail-recursion'?

   How can it help a compiler eek greater efficiency out of a recursive function in your program?

5. Given the following problem: Cover a 1xN (N >= 1) strip with 1x1 tiles or with dominoes (1x2). Find a recurrence relation T(N) for the number of ways such a strip can be covered. When N=1, a 1x1 strip can be covered only by the 1x1 tile so T(1) = 1. And a 1x2 strip can be covered by either one domino or two 1x1 tiles, so T(2) = 2. Oh, and a 1x3 strip can be covered by a single 1x1 tile and a domino or a domino and a 1x1 tile or by three 1x1 tiles so that T(3) = 3. (Note how the order mattered for the 1x3 strip.)

   Can you relate this recurrence to any other recurrence relationship we've seen? Which one? What is the relationship?

6. Write a recursive binary search routine. (Tip: A wrapper function may help keep the implementation clean.)

7. Theorem: $\sum_{i=N}^{1}((-1)^{N-i}i^2) = \frac{N(N+1)}{2}$

   Prove by induction.

8. Write a recursive function to print the elements of a singly-linked list backwards given just the head of the list.

9. Write a recursive function to insert a new value into an ordered, singly-linked list. Your only other argument should be the head of the list.

10. Use memoization to fix any performance problems that might occur with the function in problem #1 above.

11. What is the minimax concept? How does it apply to back-tracking [and games]?

    What benefits does the alpha-beta pruning technique bring to the basic minimax concept?

12. Prove the following by induction: $\frac{n+2}{n+1} + \sum_{i=1}^{n} \frac{i+1}{n+1} = \frac{n}{2} + 2$, for n >= 0.

13. Briefly explain the concept of memoization as it applies to recursive functions. (What's the basic approach? What is it trying to accomplish? When it is applicable? Etc.)

14. Find a closed form for the following recurrence relation: $U_0 = 1$; $U_n = 2U_{n-1}$; for n > 0. (Hint: Telescoping might help…)

15. Given the following table:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
J(n)	1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1

And knowing that any positive number can be written in the form $n = 2^m + r$. Find a closed form for J(n). Be sure to give all restrictions on n, m, and r that need to apply.

16. Find a closed form for the following recurrence relation: $L_0 = 1$; $L_n = L_{n-1} + n$; for $n > 0$. (Hint: Telescoping might help…)

Does the solution remind you of some other result we've shown/seen before? Which one?

17. Given that $Z_n = L_{2n} – 2n$ (for $n >= 0$), find a simplest terms closed form. (L here is the recurrence from #16.)

18. Given that a valid word in the language ABABA can contain only A's and B's and no two B's can be consecutively placed, find a recurrence relation to state the number of N letter words that can be formed in ABABA. For N=1, we have A and B, so W(1) = 2. For N=2, we have AA, AB, and BA (BB is illegal), so W(2) = 3. For N=3, we have AAA, AAB, ABA, BAA, and BAB, so W(3) = 5.

Is this recurrence related to any recurrence we've seen before? Which one(s)? What is(are) the relationship(s)?