Problem 5.2.3.9 part (a) asked us to prove for all $n \in \mathbb{N}$ that $n^2 = 2\binom{n}{2} + \binom{n}{1}$. It suggested further that we break the proof into two phases: $n < 2$ and $n \geqslant 2$.
In class we came up with a nice combinatorial proof of this equality:

> Let's say a company wants to label the items in their warehouse with 2-symbol codes. They choose the symbols for the codes from an alphabet of n symbols. Since a symbol sequence AB would be distinct from BA in this scenario, we could obviously have n^2 possible labels.
>
> For example, if $n = 3$ and the alphabet was $\{A, B, C\}$, we could form the codes:
>
> | | A | B | C |
> |---|---|---|---|
> | A | AA | AB | AC |
> | B | BA | BB | BC |
> | C | CA | CB | CC |
>
> Alternatively, we could start by choosing one symbol from the alphabet to form each of the diagonal or double-symbol codes. This can be done in $\binom{n}{1}$ ways. Then we could form the off-diagonal codes by choosing two distinct alphabet symbols which can be done in $\binom{n}{2}$ ways. But since combinations are naturally unordered, we've actually only counted part of them. Since they are 2-symbol codes, we must multiply by $2! = 2$ to account for the possible orderings of the distinct symbols. And since the double-symbol codes and the distinct-symbol codes partition the set of codes we are trying to form, we have a total of $2\binom{n}{2} + \binom{n}{1}$ codes.
>
> Since both approaches create all the possible codes for our warehouse, they must therefore be equal.

But we had a little trouble with our algebraic approach. Let's try that again.[1]
Both the book's approach and our induction approach need the $n < 2$ case handled separately. The reason is basically that both of those cases involve edge conditions for the binomial coefficients and have to be treated specially. The book's straight algebra approach, would also have an issue during cancellation with one of the values. Either way, let's do that first:

$$\begin{array}{rcl|c|rcl}
0^2 & = & 0 & \text{AND}^? & 2\binom{0}{2} + \binom{0}{1} & = & 2 \cdot 0 + 0 \\
 & & & \checkmark & & = & 0 \\
\hline
1^2 & = & 1 & \text{AND}^? & 2\binom{1}{2} + \binom{1}{1} & = & 2 \cdot 0 + 1 \\
 & & & \checkmark & & = & 1
\end{array}$$

See how the edge condition $\binom{n}{r} = 0, r > n$ came into play twice in the case 0 check and once in the case 1 check. (In addition, the case 1 check used the pseudo-edge-condition $\binom{n}{n} = 1$.)

[1] I didn't get back to it until Friday when I realized why the book had the hint about cases. But I still couldn't get our induction to work out — just straight algebra. Then, as I napped with my baby Saturday morning, the induction just unrolled in my head and I had to post this! *grin*

At this point, the book would have used this logic:[2]

$$
\begin{aligned}
2\binom{n}{2} + \binom{n}{1} &= 2\frac{n!}{2! \cdot (n-2)!} + \frac{n!}{1! \cdot (n-1)!} \\
&= 2\frac{n(n-1)}{2} + \frac{n}{1} \\
&= n(n-1) + n \\
&= n^2 - n + n \\
&= n^2
\end{aligned}
$$

All of which is legal, again, because the factors being canceled are all non-zero since $n \geqslant 2$.
Our approach, was rather, to use induction on the remaining n's. Our base case was quite straightforward, much like the $n < 2$ cases:

$$
\begin{array}{rcl|c|rcl}
2^2 & = & 4 & \text{AND}^? & 2\binom{2}{2} + \binom{2}{1} & = & 2 \cdot 1 + 2 \\
 & & & \checkmark & & = & 4
\end{array}
$$

Our inductive hypothesis was that:

$$k^2 = 2\binom{k}{2} + \binom{k}{1}$$

For some $k \geqslant 2$.

And our goal was to prove that this could lead us to having:

$$(k+1)^2 = 2\binom{k+1}{2} + \binom{k+1}{1}$$

We can now finish this induction (non-trivial steps have reasons):

$$
\begin{aligned}
(k+1)^2 &= k^2 + 2k + 1 \\
&= 2\binom{k}{2} + \binom{k}{1} + k + (k+1) && \text{inductive hypothesis \& associativity} \\
&= 2\binom{k}{2} + \binom{k}{1} + \binom{k}{1} + \binom{k+1}{1} && \text{rewrite via } \binom{n}{1} = n \text{ rule (twice)} \\
&= 2\binom{k}{2} + 2\binom{k}{1} + \binom{k+1}{1} \\
&= 2\binom{k}{2}\frac{k-1}{k-1} + 2\binom{k}{1} + \binom{k+1}{1} && \text{multiply by 1} \\
&= 2\binom{k}{1}\frac{k-1}{2} + 2\binom{k}{1} + \binom{k+1}{1} && \text{trade denominator factors} \\
&= 2\binom{k}{1}\left(\frac{k-1}{2} + 1\right) + \binom{k+1}{1} \\
&= 2\binom{k}{1}\frac{k+1}{2} + \binom{k+1}{1} \\
&= 2\binom{k+1}{2} + \binom{k+1}{1} && \text{merging factors}
\end{aligned}
$$

Note that $k + 1 - 2 = k - 1$ to make the last step complete.

[2] No, I still haven't looked at his proof. Someone wanna let me know if I guessed right? *grin*